Acoustic Wave Propagation in a Particle-Laden Gas

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In work supported by the Advanced Simulation and Computing (ASC) Turbulent Mix Project, we are continuing the development of a compressible particle/spray model for ejecta calculations [1], using the CHAD code as a test bed [2, 3].

This research uses the propagation of an acoustic wave to test the ability of the particle/spray model to compute the sound speed in a particle-laden gas. This sound speed is reduced below the sound speed of the un-laden gas. Here, a gas of uniform density ρ_g and sound speed c_g is contained in a one-dimensional tube of length L. The ends of the tube, at x = 0 and x = L, are rigid walls. The gas is laden with a uniform dispersion of small particles that exchange momentum with the gas through drag forces but do not exchange heat with the gas. The (uniform) volume fraction of the particles θ_p is small compared to unity, but, because the particle material density ρ_p is much larger than the density of the gas, the particle-phase macroscopic density $\rho_p\theta_p$ is comparable to ρ_g. The lowest sound speed of this two-phase mixture, the equilibrium sound speed c_{eq}, is attained in the limit of infinite drag between the particles and gas. Under these circumstances the sound speed c_{eq} is given by [4]

$$c_{eq}^2 = c_g^2 \frac{\rho_g}{\rho_g + \theta_p \rho_p} \tag{1}$$

Equation (1) displays the well-known lowering of the sound speed in two-phase flows. For this calculation, we used $\theta_p = 0.001$ and $\rho_p = 1000.0\rho_g$, so that for the equilibrium sound speed $c_{eq} = \frac{c_g}{\sqrt{2}}$.

Upon this uniform mixture we initially superimpose the fundamental mode velocity perturbation $u(x,0) = u_m \sin(\pi x/L)$, where u_m is the initial velocity at the center of the tube and L is the perturbation wavelength. There is no pressure perturbation, so that $u_t(x,0) = 0$. This is an acoustic perturbation, so that $u_m << c_{eq}$. The analytic solution to this problem is

$$u(x,t) = u_m \cos\left(\frac{\pi c}{L} t\right) \sin\left(\frac{\pi x}{L}\right), \tag{2}$$

where c is the mixture sound speed.

Because of numerical errors, there is numerical damping and dispersion of sound waves, and these errors are reduced by reducing both the Courant number $C = \frac{c_g \delta t}{\delta x}$, where δt is the computational time step and δx is the computational cell size, and the dimensionless wave number $k = \frac{\delta x}{L}$.

As seen in Fig. 1, for C = 0.015 and k = 0.01, the computed accuracy of the sound waves is quite acceptable. Thus, these values for the numerical parameters are used for the following results.

We performed a series of calculations in which a linear drag law was used:

$$\frac{du_p}{dt} = \frac{1}{t_{drag}}\left(u - u_p\right), \tag{3}$$

where u_p is the particle velocity and t_{drag} is a drag time that is varied from calculation to calculation. Figure 2 gives the computed mixture sound speed versus the drag time t_{drag}. The dimensional period of oscillation of the acoustic wave, based on c_g, is $P = 8.65 \times 10^{-4}$s. It is seen from Fig. 2 that the mixture sound speed is near c_{eq} for $\frac{t_{drag}}{P} > 0.1$

and near the equilibrium sound speed

$$c_{eq} \text{ for } \frac{t_{drag}}{P} < 10^{-6},$$

and drops in a monotone fashion between these values.

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[1] P.J. O'Rourke, et al., "Status of CHAD's Particle/Spray Model – September 30, 2005," Los Alamos National Laboratory report LA-UR-05-7412 (Sept. 2005).
[2] P.J. O'Rourke and M.S. Sahota, *J. Comput. Phys.* **143**, 312–345 (1998).
[3] P.J. O'Rourke and M. S. Sahota, "NO-UTOPIA: The Flow Solver for the CHAD Computer Program," Los Alamos National Laboratory report (in preparation).
[4] G.B. Wallace, *One-Dimensional Two-Phase Flow* (McGraw-Hill, New York, 1969).

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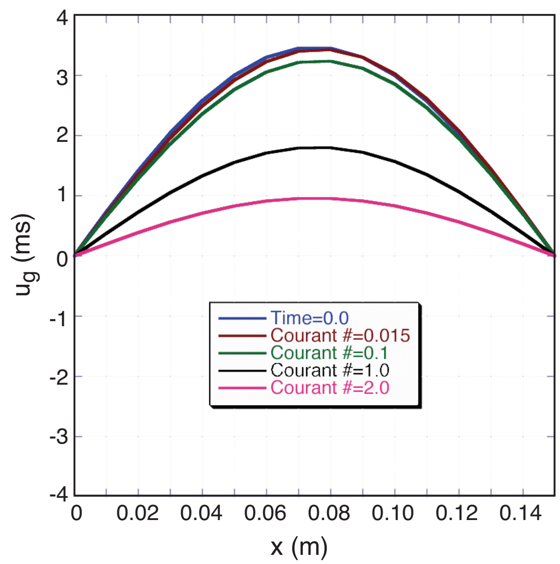

Fig. 1. *Effect of varying the Courant number on the computed solution. These are plots of gas velocity versus distance after one period of oscillation of the acoustic wave, along with the initial (time = 0.0) gas velocity profile.*

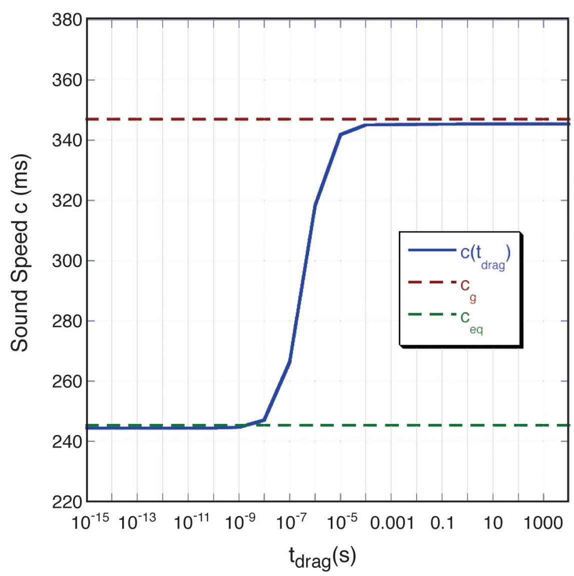

Fig. 2. *Computed mixture sound speed vs the drag time t_{drag}.*